

# NATIONAL SENIOR CERTIFICATE 

## GRADE 11

NOVEMBER 2012

## MATHEMATICS P2

MARKS: 150

TIME: 3 hours


This question paper consists of 13 pages, including 2 diagram sheets and an information sheet.

## INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions. Answer ALL the questions.
2. Clearly show ALL calculations, diagrams, graphs, et cetera, which you have used in determining the answers.
3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
4. Round off your answers to TWO decimal places if necessary, unless stated otherwise.
5. Diagrams are NOT necessarily drawn to scale.
6. Two diagram sheets for answering QUESTION 2.2, QUESTION 4.1 and 4.2, QUESTION 7.2.2 and QUESTION 11.2 are attached at the end of this question paper. Write your name on them and insert them in your answer book.
7. Number the answers correctly according to the numbering system used in this question paper.
8. Write legibly and present your work neatly.

## QUESTION 1


1.1 State whether a linear, quadratic or exponential function would best fit the data in the above scatter plot.
1.2 A researcher says that if you drive at $160 \mathrm{~km} / \mathrm{h}$, you are likely to consume more than $12 / / 100 \mathrm{~km}$. Do you agree with the researcher? Justify your answer.
1.3 What advice would you give to drivers about their driving speed in order to keep fuel consumption to the minimum?

## QUESTION 2

The following are the marks (out of 50) obtained by 10 randomly selected grade 11 learners in a test:

| 31 | 22 | 25 | 11 | 44 | 35 | 36 | 42 | 18 | 49 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

2.1 Determine the following:
2.1.1 the median
2.1.2 the semi-interquartile range
2.2 Draw a box and whisker diagram using the information in QUESTION 2.1.

Use DIAGRAM SHEET 1.
2.3 Hence, comment on the distribution of data.

## QUESTION 3

The mean age of the first 13 spectators who went to St George's Park to watch an ODI (South Africa versus Australia) cricket match is 27 . The 13 ages are given below:

$$
\begin{array}{lllllllllllll}
20 & 32 & 25 & 14 & x & 38 & 22 & 30 & 19 & 28 & 34 & 40 & 25
\end{array}
$$

3.1 Calculate the value of $x$.
3.2 Hence, determine the standard deviation for the ages.
3.3 Determine how many of the spectators had an age which is within one standard deviation of the mean.

## QUESTION 4

The following table represents the marks achieved by 65 grade 11 learners in a Mathematics test out of 40 marks:

| Interval | Frequency | Cumulative frequency |
| :---: | :---: | :---: |
| $5 \leq x<10$ | 5 |  |
| $10 \leq x<15$ | 9 |  |
| $15 \leq x<20$ | 14 |  |
| $20 \leq x<25$ | 17 |  |
| $25 \leq x<30$ | 11 |  |
| $30 \leq x<35$ | 7 |  |
| $35 \leq x<40$ | 2 |  |

4.1 Complete the cumulative frequency table using DIAGRAM SHEET 1.
4.2 Draw the ogive (cumulative frequency graph) for the above data using DIAGRAM SHEET 1.
4.3 The school decided to reward learners who obtained $80 \%$ and above. How many learners were rewarded?

## QUESTION 5

In the diagram below, STAR is a quadrilateral with vertices $S(-6 ; 4), T(-1 ; 3)$, $\mathrm{A}(p ;-17)$ and $\mathrm{R}(-7 ;-1)$.
$B$ is the midpoint of RT. SBA is a straight line.


### 5.1 Show that $\Delta \mathrm{STR}$ is isosceles.

5.2 Determine the coordinates of B, the midpoint of RT.
5.3 Determine the equation of line SA.
5.4 Hence, calculate the numerical value of $p$.
5.5 Determine whether AS is perpendicular to TR or not.
5.6 What type of quadrilateral is STAR? Give reasons for your answer.

## QUESTION 6

$\mathrm{P}, \mathrm{Q}(7 ; 6)$ and $\mathrm{R}(4 ;-6)$ are the vertices of $\triangle \mathrm{PQR}$. P is on the $x$-axis.
The equation of PR is $x+y+2=0$.
$\theta$ and $\alpha$ are the angles of inclination of PQ and QR respectively as shown in the diagram.

6.1 Determine the equation of a line parallel to PR passing through Q .
6.2 Determine the gradient of QR .
6.3 Determine the coordinates of P .
6.4 Determine the coordinates of T, if TPRQ is a parallelogram.
6.5 Determine the size of $\mathrm{P} \widehat{\mathrm{Q}}$.

## QUESTION 7

7.1 $R(6 ;-1)$ is a point on the Cartesian plane. Determine the co-ordinates of $\mathrm{R}^{\prime}$, the image of $R$, if:
7.1.1 R is rotated about the origin through $90^{\circ}$ in a clockwise direction.
7.1.2 R is reflected in the line $y=0$.
7.2 $\Delta \mathrm{DEF}$ is transformed to its image $\Delta \mathrm{D}^{\prime /} \mathrm{E}^{\prime /} \mathrm{F}^{/ /}$as follows:

- Reflection in the $x$-axis $(y=0)$,
- Followed by a translation of 3 units to the left.
7.2.1 Determine a single rule that transformed $\Delta \mathrm{DEF}$ to $\Delta \mathrm{D}^{\prime /} \mathrm{E}^{/ /} \mathrm{F}^{\prime \prime}$.
7.2.2 Hence or otherwise, draw $\Delta \mathrm{D}^{/ /} \mathrm{E}^{/ /} \mathrm{F}^{/ /}$if the vertices of $\Delta \mathrm{DEF}$ are $\mathrm{D}(4 ; 3)$,
$\mathrm{E}(0 ;-1)$ and $\mathrm{F}(5 ;-2)$. Use DIAGRAM SHEET 2 .
7.2.3 Comment on the rigidity of the transformation of $\Delta \mathrm{DEF}$ to $\Delta \mathrm{D}^{/ /} \mathrm{E}^{/ /} \mathrm{F}^{/ \prime}$.
7.3 Quadrilateral KLMN is enlarged to $\mathrm{K}^{\prime} \mathrm{L}^{\prime} \mathrm{M}^{\prime} \mathrm{N}^{\prime}$ using a scale factor of 3 .
7.3.1 Write down the coordinates of $\mathrm{N}^{\prime}$ if N is the point $\mathrm{N}(1 / 2 ;-2)$.
7.3.2 Determine the perimeter of $K^{\prime} L^{\prime} M^{\prime} N^{\prime}$ if the perimeter of KLMN is 10 units.
7.4 Describe in words the rule for rotating $\mathrm{T}(-4 ; 1)$ to $\mathrm{T}^{\prime}(-1 ;-4)$.


## QUESTION 8

The diagram below shows a new container used for oil that is to be sold at garages. The container is made up of a cylinder and a cone. The height, h , of the cylinder is 15 cm and the slant height, s , of the cone is 10 cm .
(Formulae: $\mathrm{V}=\frac{1}{3}$ area of base $\times \mathrm{H}, \quad \mathrm{V}=\pi \mathrm{r}^{2} \mathrm{~h}, \quad \mathrm{SA}=\pi \mathrm{r}^{2}+2 \pi r \mathrm{~h}, \quad \mathrm{SA}=\pi \mathrm{r} \mathrm{s}$ )

8.1 Determine the radius, $r$, if the volume of the cylinder is $4000 \mathrm{~cm}^{3}$.
8.2 Hence, determine the total volume of the container.
8.3 Calculate the total surface area of the container.

## QUESTION 9

9.1 If $\sin 29^{\circ}=p$ determine the following in terms of $p$ :
9.1.1 $\cos 29^{\circ}$
9.1.2 $\tan \left(-569^{\circ}\right)$
9.1.3 $1-\cos ^{2} 61^{\circ}$
9.2 Prove the following identity:

$$
\begin{equation*}
\left(\frac{1}{\sin \beta}+\frac{1}{\tan \beta}\right)^{2}=\frac{1+\cos \beta}{1-\cos \beta} \tag{5}
\end{equation*}
$$

## QUESTION 10

10.1 Simplify without using a calculator:

$$
\begin{equation*}
\frac{\sin (-x) \cdot \tan \left(x-360^{\circ}\right) \cdot \sin \left(450^{\circ}-x\right)}{\cos 180^{\circ}}+\cos ^{2}\left(x-180^{\circ}\right) \tag{8}
\end{equation*}
$$

10.2 Determine the general solution of:
$\sin x-3 \cos x=0$
10.3 Solve for $\alpha$ if:
2. $\sqrt{\sin \alpha}=1$ for $\alpha \in\left[90^{\circ} ; 270^{\circ}\right]$

## QUESTION 11

Given: $f(x)=-\sin x$ and $g(x)=\cos \left(x-30^{\circ}\right)$
11.1 Write down the maximum value of $3 . g(x)$.
11.2 Sketch the graphs of $f$ and $g$ on the same system of axes on DIAGRAM SHEET 2 for $x \in\left[-180^{\circ} ; 180^{\circ}\right]$.
11.3 Use your graph to determine the values of $x$, for $x \in\left[-180^{\circ} ; 180^{\circ}\right]$, for which: $g(x)-f(x) \leq 0$
11.4 Answer the following questions:
11.4.1 Write down the equation of $h$ if $h$ is the translation of $g$ by $60^{\circ}$ to the right and 1 unit up.
11.4.2 Determine the maximum value of $h(x)-f(x)$.
11.5 Explain why the reflection of $f$ in the $x$-axis and the reflection of $f$ in the $y$-axis will both result in the same graph.

## QUESTION 12

Trapezium PQRT is a plot of land bought by a farmer. RST is a straight line.
$\triangle \mathrm{QRS}$ is right-angled at R and PQST is a parallelogram.
$\mathrm{QR}=40 \mathrm{~m}, \mathrm{PS} \mathrm{T}=85^{\circ}$ and $\mathrm{Q} \widehat{\mathrm{S}} \mathrm{R}=30^{\circ}$.

12.1 Calculate the length of QS.
12.2 Calculate the length of PQ .
12.3 Determine the area of the trapezium PQRT.

## INFORMATION SHEET: MATHEMATICS

$x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$
$A=P(1+n i) \quad A=P(1-n i) \quad A=P(1-i)^{n} \quad A=P(1+i)^{n}$
$\sum_{i=1}^{n} 1=n \quad \sum_{i=1}^{n} i=\frac{n(n+1)}{2} \quad T_{n}=a+(n-1) d \quad \mathrm{~S}_{n}=\frac{n}{2}(2 a+(n-1) d)$
$T_{n}=a r^{n-1} \quad S_{n}=\frac{a\left(r^{n}-1\right)}{r-1} ; \quad r \neq 1 \quad S_{\infty}=\frac{a}{1-r} ;-1<r<1$
$F=\frac{x\left[(1+i)^{n}-1\right]}{i} \quad P=\frac{x\left[1-(1+i)^{-n}\right]}{i}$
$f^{\prime}(x)=\lim _{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$
$d=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}} \quad \mathrm{M}\left(\frac{x_{1}+x_{2}}{2} ; \frac{y_{1}+y_{2}}{2}\right)$
$y=m x+c \quad y-y_{1}=m\left(x-x_{1}\right) \quad m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \quad m=\tan \theta$
$(x-a)^{2}+(y-b)^{2}=r^{2}$
In $\triangle \mathrm{ABC}: \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \quad a^{2}=b^{2}+c^{2}-2 b c \cdot \cos A \quad$ area $\triangle A B C=\frac{1}{2} a b \cdot \sin C$
$\begin{array}{lc}\sin (\alpha+\beta)=\sin \alpha \cdot \cos \beta+\cos \alpha \cdot \sin \beta & \sin (\alpha-\beta)=\sin \alpha \cdot \cos \beta-\cos \alpha \cdot \sin \beta \\ \cos (\alpha+\beta)=\cos \alpha \cdot \cos \beta-\sin \alpha \cdot \sin \beta & \cos (\alpha-\beta)=\cos \alpha \cdot \cos \beta+\sin \alpha \cdot \sin \beta\end{array}$
$\cos 2 \alpha=\left\{\begin{array}{l}\cos ^{2} \alpha-\sin ^{2} \alpha \\ 1-2 \sin ^{2} \alpha \\ 2 \cos ^{2} \alpha-1\end{array} \quad \sin 2 \alpha=2 \sin \alpha \cdot \cos \alpha\right.$
$(x ; y) \rightarrow(x \cos \theta+y \sin \theta ; y \cos \theta-x \sin \theta) \quad(x ; y) \rightarrow(x \cos \theta-y \sin \theta ; y \cos \theta+x \sin \theta)$
$\bar{x}=\frac{\sum f x}{n}$
$\sigma^{2}=\frac{\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}}{n}$
$P(A)=\frac{n(A)}{n(S)}$
$P(A$ or $B)=P(A)+P(B)-P(A$ and $B)$

## DIAGRAM SHEET 1

NAME:


## QUESTION 2.2



## QUESTION 4.1

| Interval | Frequency | Cumulative frequency |
| :---: | :---: | :---: |
| $5 \leq x<10$ | 5 |  |
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| $15 \leq x<20$ | 14 |  |
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## QUESTION 4.2



NAME: $\square$

## QUESTION 7.2.2



QUESTION 11.2


